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Abstract

Robert Barro (1974) showed government debt has no real effects when generations are linked by altruistically motivated intergenerational transfers, a result now known widely as the Ricardian Equivalence Theorem.

An important condition for debt neutrality is believed to be the absence of strategic interactions between members of different generations. I use a simple two-period, parent and child model in which the parent is altruistic, to show Ricardian equivalence holds in the presence of intergenerational strategic behavior for a broad class of utility functions. The intuition for this result derives from the fact that the child's utility is a public good.

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I. Introduction

The Ricardian equivalence theorem presents us with a powerful result: no real redistributive effects result from deficit financing by government.[1] Substitution of debt for current taxation causes changes in private sector investment (and intergenerational transfers if necessary) that exactly offset the public transfer effects of deficit financing. Economists have examined Ricardian equivalence in a variety of contexts. Several factors have been identified that cause Ricardian equivalence to fail.[2] One factor believed to negate Ricardian equivalence, but little explored in the literature, is the presence of strategic interactions between members of different generations. This paper shows that, for a broad class of utility functions, when we allow intergenerational strategic behavior the government's financing choice has no effect on the resulting allocations. This version of the Ricardian equivalence theorem is shown to be robust with respect to details of the strategic environment. This result demonstrates Ricardian equivalence *can* hold in the presence of strategic behavior.[3] This result holds even though allowing strategic behavior can change the resulting allocations, compared to those resulting in a non-manipulative environment.

Seater (1993) writes that when strategic behavior is included in parent-child interactions "a debt-for-tax swap alters the threat point of the parents and/or the children and therefore has real effects, negating Ricardian equivalence." (p. 148). In his review of Ricardian equivalence literature he finds only a handful of authors who attempt to connect strategic behavior and Ricardian equivalence – with mixed results to date. This paper seeks to clarify that connection.

The model employed here is a version of the Samaritan's dilemma.[4] An altruistic parent makes an end-of-life transfer to a selfish child.[5] The selfish child attempts to elicit as large a transfer as possible from the parent. The parent faces the problem of how to help the selfish child without compromising his own consumption too much. Since successful manipulation by the child alters the margins at which decisions are made, the parent saves and transfers different amounts than he does without the strategic behavior. Consider the subsequent effect of government substitution of debt for taxes. It seems unlikely the parent increases his transfer by the amount of a tax decrease when confronted with a manipulative child. Thus we expect Ricardian equivalence will fail in this environment. I show that, in spite of the altered decision margins, Ricardian equivalence continues to hold in this framework.

To determine if Ricardian equivalence holds I consider two options for government financing. The first finances government expenditures through current taxation of the parent. The second finances expenditures with debt. The debt is subsequently retired through taxation of the child. Comparison of the allocations resulting from each financing option will indicate whether Ricardian equivalence holds.

Few studies of Ricardian equivalence exploit the potential for manipulation inherent in economic interactions between generations. Two exceptions are Bruce and Waldman (1990) and Kotlikoff, Razin, and Rosenthal (1990). Both use static models with parental altruism to evaluate the effects of public transfer programs. Bruce and Waldman use a structure similar to the model of this paper but allow parent-to-child transfers in both periods. The child can manipulate the second period transfer by overconsuming in the first period. The parent can mitigate this manipulation by making a first period transfer, which reduces his ability to make a second period transfer. The authors point out that, as the size of the first period transfer increases, there is a point at which the second period transfer becomes inoperable, and the child no longer overconsumes in the first period. In this environment, public parent-to-child transfers are equivalent to private first period transfers. As we consider successively larger public transfers we eventually reach the point at which the second period transfer becomes inoperable. At this point additional public transfers are no longer fully offset by a corresponding decrease in private transfers. One might be tempted to conclude that the failure of Ricardian equivalence in their model is somehow related to the strategic behavior of the child. Closer inspection reveals that the failure of Ricardian equivalence is in fact due to the elimination of an operative transfer motive and not to the presence of intergenerational strategic behavior. Interestingly, if we instead consider public child-to-parent transfers (as is government issuance of debt), public transfers are always fully offset by changes in private transfers in their framework.

To examine the relationship between strategic behavior and Ricardian equivalence it is important to eliminate other factors which may negate Ricardian equivalence. The existence of intergenerational transfers both before and after a policy change is a well documented requirement for Ricardian equivalence. Barro (1989) summarizes this requirement as follows.

> First, intergenerational transfers do not have to be "large;" what is necessary is that transfers based on altruism be operative at the margin for most people. Specifically, most people must be away from the corner solution of zero transfers, where they would, if permitted, opt for negative payments to their children. (The results go through, however, if children typically support their aged parents.) (p. 41)

Kotlikoff, Razin, and Rosenthal (1990) use a model with two-way altruism. Private transfers are allowed in either direction. The unique aspect of this model is that an individual may refuse a gift if he considers it to be "too small." This leads to a range of possible endowment distributions for which no private transfers occur. They observe it is possible to have a public transfer that shifts the endowments either into or out of this no-private-transfer range. Therefore, since such public transfers may not be fully offset by changes in private transfers, Ricardian equivalence fails in this framework. It is unclear, in their highly stylized model, whether Ricardian equivalence fails in general because of strategic behavior or whether this is an artifact of their model.

Both Bruce and Waldman (1990) and Kotlikoff, Razin and Rosenthal (1990) present models with intergenerational strategic behavior in which Ricardian equivalence fails. While strategic behavior can influence the point at which private transfers start (or stop), it is not clearly the cause of the failure of Ricardian equivalence. This paper shows Ricardian equivalence is not necessarily affected by allowing strategic behavior between generations. I demonstrate the robustness of this result by considering different possible sequences of actions by parent and child. Ricardian equivalence holds in each case. This shows the result is independent of the details of the environment in which strategic behavior occurs.[6]

Using a Samaritan's dilemma-like framework is not new for policy studies. Coate (1995), Bruce and Waldman (1991) and Lindbeck and Weibull (1988) all use a similar model. Each casts the government as a representative of wealthy individuals (i.e., good Samaritans).[7] Each then examines the policy implications when recipients of public transfers engage in strategic behavior. Each concludes including strategic behavior in the analysis leads to more efficient policy choices.

The results of this paper stem in part from the public good character of the child's welfare. Here, parent and child simultaneously benefit from the child's consumption without possibility of exclusion. Bergstrom, Blume and Varian (1986) and Warr (1983) show voluntary contributions to a public good are unaffected by a relatively small income redistribution amongst contributors. In effect, a change in the timing of lump sum taxes is identical to an income redistribution across generations.

The rest of the paper proceeds as follows. Section II describes the model employed and an equilibrium. Section III gives the primary Ricardian equivalence result. Section IV describes two alternative model specifications and presents their corresponding Ricardian equivalence results. Section V presents two examples using a CES period utility function. Section VI concludes the paper. Proofs of the theorems can be found in the appendices.

II. The Model

The model used for this analysis is a two-consumer two-period model with one-sided altruism. The two consumers (denoted Parent and Kid) are each alive for both periods.

The goal is to demonstrate, as generally as possible, that Ricardian equivalence always holds in this framework. To accomplish this I present proofs for several theorems:[8]

1. A unique, pure strategy equilibrium with simultaneous consumption and savings choices always exists.

2. Changing the timing and statutory incidence of a lump sum tax has no real effects when consumption and savings choices are simultaneous.

3. Changing the timing and statutory incidence of a lump sum tax has no real effects when the parent chooses consumption and savings amounts first.

4. Changing the timing and statutory incidence of a lump sum tax has no real effects when the kid chooses consumption and savings amounts first.

The second theorem provides the main result of the paper. The third and fourth theorems augment this result by demonstrating its robustness to changes in the sequence of consumption and savings choices.

A specific example may help illustrate the interactions modeled here. Consider a parent and a young child, interacting at the beginning and end of a week. At the beginning of the week the family receives its income, divided in some way between parent and kid. (Perhaps the parent receives his wage and the kid gets an allowance.) For most of the week parent and kid have little interaction – they may spend the majority of their time at work and school respectively. During this time each decides what portion of their income to spend for current consumption and what portion to set aside for consumption during the upcoming weekend.

The kid must decide how willing he is to forgo current consumption to save for consumption during the weekend. The kid recognizes he will see the parent during the weekend and that the parent likes to have a 'happy' kid. Thus if the kid spends a large portion of his income during the week and presents himself to the parent as poor, the parent is likely to give the kid more money. In fact, the more money the kid saves the less he is likely to get from the parent.[9] A manipulative kid recognizes this opportunity to extract additional funds from the parent and acts accordingly.

The child's ability to successfully manipulate the parent depends on the parent's affinity for the child and on the parent's wealth level. The child's interest in being manipulative depends primarily on his substitution rate between current and future consumption. Finally, the parent anticipates the child's manipulation and prepares for it by saving less (or perhaps even more) than he otherwise might have.

II.A. Details of the Model

There are two consumers who are alive for both of two time periods. One consumer (denoted P for parent) is altruistic towards the other consumer (denoted K for kid).[10] Each individual j is endowed with wealth w^j in their first period of life, $j = P, K$. This wealth can either be consumed or put into savings. Each unit put into savings in the first period returns $1 + r$ units in the second period. The net return on savings (r) is exogenous.

Let c_t^j denote individual j's consumption in period $t(= 1, 2)$. The kid's utility function is $U^K(c_1^K, c_2^K)$. The parent's utility function is $U^P(c_1^P, c_2^P, U^K)$. To allow separate descriptions for the utility maximization problem each consumer faces in each time period I assume the utility functions U^P and U^K are additively separable in their arguments. Then we have

$$U^K(c_1^K, c_2^K) \equiv u_1^K(c_1^K) + \beta u_2^K(c_2^K) \tag{1}$$

and

$$U^P(c_1^P, c_2^P, U^K) \equiv u_1^P(c_1^P) + \beta u_2^P(c_2^P) + \rho U^K \tag{2}$$

where $u_t^j(\cdot)$ gives consumer j's period t utility of consuming c_t^j for $j = P, K$; $t = 1, 2$. $\beta \in (0, 1]$ is the consumer's intertemporal discount rate. $\rho > 0$ is the parent's intergenerational discount rate.

Assume u_t^j has all the standard properties of utility functions for $j = P, K$; $t = 1, 2$. Specifically, each $u_t^j : \mathcal{R}_+ \to \mathcal{R}$ is thrice continuously differentiable, strictly concave and increasing and $\partial u_t^j / \partial c \to \infty$ as $c \to 0$. Thus U^j are thrice continuously differentiable and strictly concave for $j = P, K$. Assume all goods are normal for each consumer.

The parent can transfer any nonnegative amount of wealth to the kid in the second period.[11] Assume the kid cannot borrow against possible future transfers.[12] There is perfect information and no individual or aggregate uncertainty.

Now turn to the timing of the model. There exist two possible approaches. First, the parent may precommit to a transfer amount by choosing this amount in the first period. This approach has three advantages. It prevents manipulation by the kid, is relatively easy to compute, and, presumably satisfies Ricardian equivalence. We expect Ricardian equivalence to hold here because the parent bases his transfer decision on their combined endowments rather than on their combined second period wealth. A significant disadvantage of this approach is the lack of time consistency on the part of the parent in the second period.[13] The kid may choose an action in the first period that makes the parent's previously chosen transfer amount sub-optimal in the second period. This problem makes precommitment a difficult assumption to defend in most analyses.

In the second approach the parent chooses only a consumption amount in the first period and chooses the transfer amount in the second period. While providing a time consistent solution, this approach leaves the parent vulnerable to manipulation of his transfer choice by the kid's first period choices. It is believed the potential for manipulation leads to a failure of Ricardian equivalence.[14] This paper employs this second approach.

In the first period, assume both individuals choose their consumption and savings amounts simultaneously. (Section IV evaluates alternative first-period choice sequences.) In the second period the parent chooses his consumption and transfer amounts first. The kid receives the transfer and chooses his consumption amount last.

Let s^j denote the amount consumer j puts into savings. T denotes the amount the parent transfers to the kid in the second period. The period budget constraints for the parent are

$$c_1^P + s^P \leq w^P \tag{3}$$

$$c_2^P + T \leq s^P(1+r). \tag{4}$$

The period budget constraints for the kid are

$$c_1^K + s^K \leq w^K \tag{5}$$

$$c_2^K \leq s^K(1+r) + T. \tag{6}$$

The sequential nature of the model allows the use of backwards induction to compute a solution. With backwards induction we address the second period first. The kid chooses last in the second period. His problem at that time is

$$\max_{c_2^K} u_2^K(c_2^K)$$

subject to (6) and $c_2^K \geq 0$.

Since u_2^K is strictly increasing (and T and s^K are chosen before the kid makes his second period choice), the solution to this problem is simply

$$c_2^K = s^K(1+r) + T. \tag{7}$$

In fact, the strictly positive marginal utilities for both consumers imply that, in equilibrium, each budget constraint is satisfied with equality. In what follows I also assume the equilibrium transfer amount (T) is strictly positive. This stems from wanting to study only the effects of strategic behavior and not the effects of possible corner solutions – which are known to negate Ricardian equivalence.

Continuing the backwards induction we next look at the parent's second period problem:

$$\max_{c_2^P, T} u_2^P(c_2^P) + \rho u_2^K(c_2^K)$$

subject to (4) and $c_2^P, T > 0$, given (7).

Substituting all pertinent constraints into the parent's second period problem allows us to define the following function:

$$T(s^P, s^K) \equiv \underset{T}{\text{argmax}} \left[u_2^P(s^P(1+r) - T) + \rho u_2^K(s^K(1+r) + T) \right] \tag{8}$$

$$\text{such that } T > 0.$$

Recall s^P and s^K are chosen in the first period and thus are known when T is chosen. Implicit in this function is the fact that there is a unique T which maximizes the parent's second period problem. While not explicitly demonstrated here, this is not difficult to show given the strict concavity of u_2^P and u_2^K. Furthermore, $T(s^P, s^K)$ is continuous and twice differentiable.[15] This function will be useful when determining the optimal first period choices.

Given w^j the choice of s^j uniquely determines c_1^j for $j = P, K$ from equations (3) and (5) respectively. Similarly, knowing s^P and s^K allows unique determination of T using equation (8). Then, since r is exogenous, c_2^P and c_2^K are uniquely determined from equations (4) and (6) respectively. Thus the final allocations of both periods of the model are completely specified by the first period choices of s^P and s^K.

Finally, we identify the strategy spaces and payoffs for the individuals.

Definition: Let $\mathcal{S}^j = [0, w^j]$ be the space of possible savings amounts for individual $j(= P, K)$. These are the strategy spaces for the parent and kid respectively.

Payoffs are given by the respective utility functions.

II.B. Equilibrium

This section defines a Simultaneous-Choice equilibrium of the model.

Definition: A Simultaneous-Choice equilibrium of this model is a savings pair
$(\tilde{s}^P, \tilde{s}^K) \in \mathcal{S}^P \times \mathcal{S}^K$ such that:

1. $\tilde{s}^K$ solves the kid's first period problem given $\tilde{s}^P$:

$$\max_{s^K} \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(\tilde{s}^P, s^K))\Big], \tag{9}$$

2. $\tilde{s}^P$ solves the parent's first period problem given $\tilde{s}^K$:

$$\begin{aligned}\max_{s^P} \; &\Big[u_1^P(w^P - s^P) + \beta u_2^P(s^P(1+r) - T(s^P, \tilde{s}^K)) \\ &+\rho[u_1^K(w^K - \tilde{s}^K) + \beta u_2^K(\tilde{s}^K(1+r) + T(s^P, \tilde{s}^K))]\Big],\end{aligned} \tag{10}$$

3. $T(s^P, s^K)$ is as defined in equation (8) for all $(s^P, s^K) \in \mathcal{S}^P \times \mathcal{S}^K$.

In addition, define the best response functions for the parent and kid respectively as follows:

$$\begin{aligned}f^P(s^K) \equiv \underset{s^P}{\text{argmax}} \; &\Big[u_1^P(w^P - s^P) + \beta u_2^P(s^P(1+r) - T(s^P, s^K)) \\ &+\rho[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(s^P, s^K))\Big]\end{aligned} \tag{11}$$

and

$$f^K(s^P) \equiv \underset{s^K}{\text{argmax}} \; \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(s^P, s^K))\Big]. \tag{12}$$

It can be shown $f^P(\cdot)$ is continuously differentiable for all utility functions satisfying the specifications given in section II.A by applying the implicit function and envelope theorems.

Assumption A.1: $f^K(s^P)$ is continuously differentiable.

Lemma 1: Assumption A.1 holds for standard utility functions, such as CES, negative exponential, hyperbolic absolute risk aversion, and functions with constant absolute risk aversion.

That Lemma 1 holds is easily demonstrated algebraically.[16]

Theorem 1: *Given assumption A.1, a unique Simultaneous-Choice equilibrium (in pure strategies) exists.*

A proof is given in Appendix A.

III. Ricardian Equivalence

The Ricardian equivalence theorem asserts that changing the timing of taxes has no real effect on the distribution of resources. To examine this theorem in the two-period, two-consumer setting I contrast the effects of two possible tax policies. The first imposes a lump-sum tax of τ on the parent in the first period. The second imposes a lump-sum tax of $\tau(1+r)$ on the kid in the second period.[17] The larger second period tax reflects the interest that accumulates when government uses deficit financing. Instead of thinking about imposing one policy or the other, we should view this as substitution of the latter policy for the former and ask what effect this substitution has on the distribution of resources.

In the context of this model, Ricardian equivalence predicts that, when faced with a reduction of his own taxes and a corresponding increase in his kid's taxes, the parent increases the size of his transfer to help the kid with his new tax burden. As mentioned earlier, this assertion has been questioned in models that allow the parent and kid to behave strategically.

III.A. The Neutrality of Changing Statutory Tax Incidence

Theorem 2: *Take assumption A.1 as given and assume we are initially in a Simultaneous-Choice equilibrium. Consider a change in policy from a lump-sum tax of τ on the parent in the first period to a lump-sum tax of $\tau(1+r)$ on the kid in the second period. After the policy change there exists a new Simultaneous-Choice equilibrium in which both individuals consume the same amounts as they did before the policy change.*

A proof is presented in Appendix B. An example using a CES period utility function is presented in section V.A.

III.B. Discussion

The policy change is analogous to a redistribution of wealth from kid to parent. When choosing savings amounts both parent and kid know how second period wealth will be divided via the transfer function. The kid faces a new tax and realizes the parent's second period wealth has increased by the amount of the tax. Thus aggregate second period wealth is unchanged. Significant here is that the redistribution causes the parent to increase his transfer by the amount of the redistribution. Then, since the kid's first period endowment is unchanged, he effectively perceives the same resource constraint as before the policy change. Thus his optimal choice after the redistribution will be identical to the choice made before the redistribution. The argument is similar for the parent. He effectively maximizes the family's utility subject to a family budget constraint. He considers the kid's wealth, as well as his own, when choosing his savings and transfer amounts. The redistribution of wealth from kid to parent does not change the family's total wealth, so he too effectively perceives an unchanged budget constraint. Again the optimal choice after redistribution will be identical to the choice made before redistribution.

IV. Alternative First Period Choice Sequences

To ensure the result of the previous section is not simply an outcome of simultaneous first period choices this section considers alternative first period choice sequences. Two other possible sequences exist: a) the parent chooses first in the first period; and, b) the kid chooses first in the first period. Aside from this change in the order of first period choices all other details of the model remain as described in section II.A. Our uncertainty about the true nature of parent-child interactions, coupled with the fact that these sequential choice specifications produce different distributions than does the simultaneous choices specification, strongly indicates we should evaluate these as well. In this section I show Ricardian equivalence holds in each case. Evaluation of these two alternative first period choice sequences helps demonstrate the robustness of the above debt neutrality result.

IV.A. Sequential Choices with Parent Choosing First

In contrast to simultaneous choices, perhaps a more natural sequence entails the parent making his choices first. This section describes the subgame perfect equilibrium resulting when the parent chooses first in the first period. This equilibrium is referred to as the "Parent-First equilibrium."

IV.A.a Equilibrium

The strategy space and payoffs are as follows.

Definition: Let $\mathcal{S}^j = [0, w^j]$ be the space of possible savings amounts for individual $j (= P, K)$. These are the strategy spaces for the parent and kid respectively.

Payoffs are given by the respective utility functions.

A Parent-First equilibrium of this model is a savings pair $(\tilde{s}^P, \tilde{s}^K) \in \mathcal{S}^P \times \mathcal{S}^K$ such that:

1. $\tilde{s}^K$ solves the kid's first period problem given $\tilde{s}^P$:

$$\max_{s^K} \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(\tilde{s}^P, s^K))\Big], \tag{13}$$

   Let the solution to this problem be denoted $s^K(s^P)$.

2. $\tilde{s}^P$ solves the parent's first period problem:

$$\begin{aligned} \max_{s^P} \Bigg[& u_1^P(w^P - s^P) + \beta u_2^P\Big(s^P(1+r) - T(s^P, s^K(s^P))\Big) \\ & +\rho\Big[u_1^K(w^K - s^K(s^P)) + \beta u_2^K\Big(s^K(s^P)(1+r) + T(s^P, s^K(s^P))\Big)\Big]\Bigg], \end{aligned} \tag{14}$$

3. $T(s^P, s^K)$ is as defined in equation (8) for all $(s^P, s^K) \in \mathcal{S}^P \times \mathcal{S}^K$.

Because this is a finite horizon model with complete information, the use of backwards induction assures existence of a subgame perfect Parent-First equilibrium. The strictly concave utility functions assure no ties in the payoffs; thus the equilibrium is unique.

IV.A.b Ricardian Equivalence

Theorem 3: *Take assumption A.1 as given and assume we are initially in a Parent-First equilibrium. Consider a change in policy from a lump-sum tax of τ on the parent in the first period to a lump-sum tax of $\tau(1+r)$ on the kid in the second period. After the policy change there exists a new Parent-First equilibrium in which both individuals consume the same amounts as they did before the policy change.*

A proof is presented in Appendix C.

The intuition for this result is similar to that for the simultaneous choice specification. The parent acts first and knows how the kid will react to his choices. Thus the parent effectively determines the distribution of the total endowment of the family. This total endowment is unchanged by a change in tax policy so the resulting allocations are unchanged as well.

IV.B. Sequential Choices with Kid Choosing First

Although less likely than the two options considered so far, it is possible the kid would choose first in the first period. This section describes the resulting subgame perfect equilibrium, which is referred to as the "Kid-First equilibrium."

IV.B.a Equilibrium

The strategy space and payoffs are as follows.

Definition: Let $\mathcal{S}^j = [0, w^j]$ be the space of possible savings amounts for individual $j(= P, K)$. These are the strategy spaces for the parent and kid respectively.

Payoffs are given by the respective utility functions.

A Kid-First equilibrium of this model is a savings pair $(\tilde{s}^P, \tilde{s}^K) \in \mathcal{S}^P \times \mathcal{S}^K$ such that:

1. $\tilde{s}^P$ solves the parent's first period problem given $\tilde{s}^K$:

$$\max_{s^P} \Big[u_1^P(w^P - s^P) + \beta u_2^P(s^P(1+r) - T(s^P, \tilde{s}^K))$$

$$+\rho\Big[u_1^K(w^K - \tilde{s}^K) + \beta u_2^K\Big(\tilde{s}^K(1+r) + T(s^P, \tilde{s}^K)\Big)\Big]\Big], \tag{15}$$

Let the solution to this problem be denoted $s^P(s^K)$.

2. $\tilde{s}^K$ solves the kid's first period problem:

$$\max_{s^K} \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(s^P(s^K), s^K))\Big], \tag{16}$$

3. $T(s^P, s^K)$ is as defined in equation (8) for all $(s^P, s^K) \in \mathcal{S}^P \times \mathcal{S}^K$.

Because this is a finite horizon model with complete information, the use of backwards induction assures existence of a subgame perfect Kid-First equilibrium. The strictly concave utility functions assure no ties in the payoffs; thus the equilibrium is unique.

IV.B.b Ricardian Equivalence

Theorem 4: *Take assumption A.1 as given and assume we are initially in a Kid-First equilibrium. Consider a change in policy from a lump-sum tax of τ on the parent in the first period to a lump-sum tax of $\tau(1+r)$ on the kid in the second period. After the policy change there exists a new Kid-First equilibrium in which both individuals consume the same amounts as they did before the policy change.*

A proof is presented in Appendix D.

The kid acts first and knows how the parent will react to his choices. Thus the choices of the kid determine the entire distribution of the family's endowment. Since the total endowment is unchanged by a change in tax policy the resulting allocations are unchanged as well.

V. Two Examples

This section presents examples of the foregoing arguments. The first subsection contains an example showing algebraically that Ricardian equivalence holds for a utility function which is both tractable and reasonably robust (CES). The example is for the case of simultaneous first period choices. The second subsection presents an example quantifying the differences between the equilibria of the different possible choice sequences. I choose a set of parameter values and calculate and compare the different equilibria. The intent here is to clarify the impact of the alternative model specifications.

To begin I describe some features common to both examples. Each uses a common CES period utility function. That is, $u_t^j(c) = \frac{c^\gamma}{\gamma}$ for $j = P, K; t = 1, 2$. Then the kid's and parent's total utilities are as follows:

(17) $$U^K = \frac{(c_1^K)^\gamma}{\gamma} + \beta \frac{(c_2^K)^\gamma}{\gamma}$$

(18) $$U^P = \frac{(c_1^P)^\gamma}{\gamma} + \beta \frac{(c_2^P)^\gamma}{\gamma} + \rho U^K.$$

with $\gamma < 1$, $\gamma \neq 0$, $0 < \beta \leq 1$, and $\rho > 0$.

Let τ^P denote the amount of a lump-sum tax imposed on the parent in the first period. Let τ^K denote the amount of a lump-sum tax imposed on the kid in the second period.

The budget constraints for the parent and kid respectively for each period are

$$c_1^P = w^P - s^P - \tau^P$$

$$c_2^P = s^P(1 + r) - T$$

and

$$c_1^K = w^K - s^K$$

$$c_2^K = s^K(1 + r) + T - \tau^K.$$

The first order condition for the parent's second period transfer is

$$u'(c_2^P) - \rho u'(c_2^K) = 0.$$

Using CES utility functions this becomes

$$c_2^P \rho^{\frac{1}{1-\gamma}} = c_2^K. \tag{19}$$

Substituting in for the second period consumption amounts and solving for T gives

$$T(s^P, s^K) = \frac{s^P \rho^\alpha (1+r) - s^K(1+r) + \tau^K}{1+\rho^\alpha}, \tag{20}$$

where $\alpha = \frac{1}{1-\gamma}$.

Thus

$$\frac{\partial T}{\partial s^P} = \frac{\rho^\alpha (1+r)}{1+\rho^\alpha} \tag{21}$$

and

$$\frac{\partial T}{\partial s^K} = -\frac{1+r}{1+\rho^\alpha}. \tag{22}$$

V.A. A Simultaneous-Choice Example using CES Utility Functions

This section examines a Simultaneous-Choice equilibrium for the example of CES period utility functions. I contrast the effects of two possible government financing policies. The first policy imposes a lump sum tax of τ units on the parent in the first period; i.e., $(\tau^P, \tau^K) = (\tau, 0)$. The second policy imposes a lump sum tax of $\tau(1+r)$ on the kid in the second period; i.e., $(\tau^P, \tau^K) = (0, \tau(1+r))$. To simplify exposition of these examples I assume an interior solution for s^K. This assumption has no effect on the Ricardian equivalence results of these examples, a claim substantiated by the preceding theorems.

When choosing a savings amount the parent takes the kid's savings choice as given. Thus his first order condition for saving is

$$u'(c_1^P) = \beta u'(c_2^P)(1 + r - \frac{\partial T}{\partial s^P}) + \rho\beta u'(c_2^K)\frac{\partial T}{\partial s^P}. \tag{23}$$

Using the CES utility functions and equations (20) and (21), substituting in for the consumption amounts and solving for s^P gives

$$s^P = \frac{(w^P - \tau_P)(1+\rho^\alpha)[\beta(1+r)]^\alpha - s^K(1+r) + \tau^K}{(1+\rho^\alpha)[\beta(1+r)]^\alpha + 1 + r}. \tag{24}$$

When choosing a savings amount the kid takes the parent's savings choice as given. Thus his first order condition for savings is

$$u'(c_1^K) = \beta u'(c_2^K)\Big(1 + r + \frac{\partial T}{\partial s^K}\Big). \tag{25}$$

Using the CES utility functions and equations (20) and (22), substituting in for the consumption amounts and solving for s^K gives

$$s^K = \frac{[\beta\rho^\alpha(1+r)]^\alpha(1+\rho^\alpha)^{1-\alpha}w^K - s^P\rho^\alpha(1+r) + \tau^K\rho^\alpha}{\rho^\alpha(1+r) + [\beta\rho^\alpha(1+r)]^\alpha(1+\rho^\alpha)^{1-\alpha}}. \tag{26}$$

Using the above expressions for s^P, s^K and T allows expression of the consumption amounts in terms of w^P, w^K, τ^P, τ^K, r, β, and ρ as follows.

$$\begin{aligned}
c_1^P &= [(w^P + w^K - \tau^P)(1+r) - \tau^K][\beta\rho^\alpha(1+r)]^\alpha(1+\rho^\alpha)^{1-\alpha}A_1\\
c_2^P &= [(w^P + w^K - \tau^P)(1+r) - \tau^K][\beta\rho^\alpha(1+r)]^\alpha[\beta(1+r)]^\alpha(1+\rho^\alpha)^{1-\alpha}A_1\\
c_1^K &= [(w^P + w^K - \tau^P)(1+r) - \tau^K][\beta(1+r)]^\alpha\rho^\alpha(1+\rho^\alpha)A_1\\
c_2^K &= [(w^P + w^K - \tau^P)(1+r) - \tau^K]\rho^\alpha[\beta\rho^\alpha(1+r)]^\alpha[\beta(1+r)]^\alpha(1+\rho^\alpha)^{1-\alpha}A_1
\end{aligned}$$

where

$$\begin{aligned}
A_1 = \Big(&\rho^\alpha(1+r)(1+\rho^\alpha)[\beta(1+r)]^\alpha + (1+r)[\beta\rho^\alpha(1+r)]^\alpha(1+\rho^\alpha)^{1-\alpha}\\
&+[\beta(1+r)]^\alpha[\beta\rho^\alpha(1+r)]^\alpha(1+\rho^\alpha)^{2-\alpha}\Big)^{-1}.
\end{aligned}$$

Compare the consumption amounts under the two possible tax policies. Recall either $\tau^P = \tau$ or $\tau^K = \tau(1+r)$. Clearly all quantities are the same under each policy.

V.B. A Numerical Example

This section presents a numerical example comparing the equilibria of the different specifications of the model. It also offers some discussion on the differences between the equilibria. As for the preceding example, I again assume a common CES period utility function.

The specific parameter values for this example are given in Table 1. These parameters were chosen arbitrarily and have no particular economic significance.[18] They provide comparative results representative of the many different parameterizations evaluated.

Table 2 presents the allocations, transfer amount and utilities for the equilibria of each different specification. In each case the lump sum tax τ is collected from the parent in the first period.[19]

To begin I contrast precommitment with the simultaneous choice specification. The claim made in the introduction of this paper was that manipulation allows the kid to squander resources early in life so as to extract a larger transfer from the parent later. The first two columns of Table 2 present results for the precommitment and simultaneous choice regimes respectively and show that, when manipulation is allowed, the kid's first period consumption increases while second period consumption decreases. Correspondingly, the parent makes a larger transfer and the kid experiences a net utility increase. This illustrates squandering early in life improves the kid's total utility.

The last two columns of Table 2 present corresponding results for the Parent-First and Kid-First specifications respectively. Comparing the three manipulative specifications we see the parent has a greater utility in the Parent-First specification than in the simultaneous choice specification. This reflects his ability to somewhat mitigate the manipulation of the kid by choosing first. He accomplishes this by increasing his own first period consumption, thereby saving less for the second period for the kid to attempt to extract from him. In the Kid-First specification we see a larger transfer and a greater utility for the kid than in either the simultaneous choices or Parent-First specifications. This reflects the kid's increased ability to manipulate the parent when moving first.

Table 1: Parameter Values

Parameter	Value
β	0.7
γ	-2
ρ	0.6
w^P	12
w^K	8
r	$1/\beta - 1$
τ	2

Table 2: Comparison of Alternative Model Specifications

	Precommitment	Simultaneous Choices Specification	Parent-First Specification	Kid-First Specification
c_1^P	5.746	5.318	5.682	4.978
c_2^P	5.746	5.318	5.165	4.978
c_1^K	4.843	5.821	5.653	6.598
c_2^K	4.843	4.485	4.356	4.199
T	0.332	1.371	1.004	2.195
U^P	-0.04749	-0.04935	-0.04906	-0.05310
U^K	-0.03624	-0.03216	-0.03409	-0.03134

VI. Conclusion

This paper demonstrates that allowing strategic behavior between different generations does not necessarily negate Ricardian equivalence. In a static, two-period model a kid may attempt manipulation of the size of a transfer given him by his parent. I show Ricardian equivalence holds in this framework regardless of the sequence of actions employed.

We can extend the result to a policy shift from a second period tax on the kid to a first period tax on the parent – a public transfer from parent to kid. One additional stipulation required is that the new tax on the parent cannot exceed his initial savings amount. These two results together can be used to demonstrate the neutrality of a range of policy options including deficit financing and social security programs.

These results need not be surprising given the literature on voluntary contributions to a public good. Bergstrom, Blume and Varian (1986) and Warr (1983) demonstrate that a wealth redistribution amongst contributors to a public good has no effect of the provision of the public good. In my model the public good is the kid's utility. Parent and kid both enjoy the kid's utility non-rivalrously and without possibility of exclusion. Thus we could expect a change in the timing of taxes to have no effect on the final consumption amounts.

The results are also consistent with Varian's (1994) study of private provision of public goods. He compares public good provision when contribution choices are simultaneous or sequential. He shows underprovision of a public good results when the individual valuing it most chooses his contribution first. This underprovision is relative to the amount provided under simultaneous choices.

To apply Varian's (1994) result, first distinguish between the kid's first and second period consumption by recalling that the parent only directly affects the kid's second period consumption. Then the kid's second period consumption is the public good to which both individuals make voluntary contributions – the parent via transfer and the kid via savings. The kid's first period consumption merely provides a positive externality for the parent.

In my model the kid values the public good most (assuming $\rho < 1$). In the manipulative framework he makes his contribution choice (i.e., his savings amount) first. In precommitment both individuals choose their contributions simultaneously. The numerical example of the previous section shows, as Varian (1994) predicts, the kid's second period consumption is lower under the manipulative regime than under precommitment.

Two extensions of this result naturally arise. First, it is likely this debt neutrality result also holds in dynamic environments. It should be possible to construct a revealed preference argument for infinitely lived families as was done for those modelled here. Additional work is already underway to verify this claim. Second, we can no longer rely on altered decision margins to indicate when Ricardian equivalence will or will not hold. In light of this fact we should reexamine arguments that appeal simply to altered decision margins to indicate failure of Ricardian equivalence.

It is perhaps tempting to conclude further studies of the effects of deficit financing need not include strategic behavior. However, we observe that allowing strategic behavior does change the resulting allocations, compared to a non-manipulative regime. Thus it seems important to continue considering strategic behavior in our analyses. Additional empirical analysis, to determine a value for the strength of intergenerational altruism, would help clarify this issue.

Notes

[1]Although Ricardo (1819) was the first to postulate the neutrality of government debt, he also considered such neutrality unlikely, advising that "it must not be inferred that I consider the system of borrowing as the best calculated .... It is a system which tends to make us less thrifty – to blind us to our real situation." (p. 303-304).

[2]See Barro (1989), Seater (1993) and Leiderman and Blejer (1988) for surveys of the required conditions and their respective significance. Barro (1989) and Seater (1993) also provide reviews of the micro and macroeconomic studies that test for empirical evidence of Ricardian equivalence.

[3]This paper shows Ricardian equivalence holds in the presence of one possible form of intergenerational strategic behavior. The intent is not to assert Ricardian equivalence holds for all forms of strategic behavior, but rather to prove intergenerational strategic behavior does not necessarily cause Ricardian equivalence to fail.

[4]The "Samaritan's dilemma" was introduced by James M. Buchanan (1975). He observed that an altruist who sacrificed part of his own consumption potential to help another faced the dilemma of how much aid to render.

[5]Bernheim, Shleifer and Summers (1985) suggest a desire for child-to-parent services (e.g. phone calls, frequent visits, etc.), rather than altruism, motivates parental transfers. The true motivation for parent-to-child transfers remains an open question in economics and is not an issue addressed in this paper. Bernheim (1991) offers additional discussion on this question. The evidence seems to weigh more heavily in favor of altruism hence it is the motivation used in this paper.

[6]This is true in spite of the fact that altering the sequence of actions *does* change the resulting allocations.

[7]Lindbeck and Weibull (1988) point out some of the other applications which exist.

[8]These theorems are shown to hold for standard forms of utility functions, such as CES, negative exponential, and HARA, and all functions with constant absolute risk aversion.

[9]Some would say a kid who overconsumes when young is just shortsighted. In fact, a kid who knows his parent will not let him suffer is acting rationally by overconsuming in an early period.

[10]Identifying the consumers as parent and kid is not meant to restrict application of the results to intrafamily interactions. For example, Coate (1995) uses a similar framework to evaluate the efficiency of public transfers from rich to poor individuals.

[11]Assuming no transfer occurs in the first period merely simplifies the analysis but has no effect on the results.

[12]This constraint is motivated by the observation that, in practice, it is difficult to borrow against a potential future bequest.

[13]An equivalent description is to say the resulting equilibrium lacks subgame perfection on the part of the parent with this approach.

[14]As in Kotlikoff, Razin and Rosenthal (1990) and Bruce and Waldman (1990).

[15]Continuity of $T(s^P, s^K)$ comes from applying the theorem of the maximum and using the fact that u_2^P and u_2^K are continuous and strictly concave. Twice differentiability comes from application of the implicit function theorem and the thrice differentiability of u_2^P and u_2^K.

[16]Specifically, each consumer may have a different utility parameter(s) for each time period. Then the following conditions suffice for lemma 1 to hold:

- CES utility functions ($u_t^j(c_t^j) = \frac{(c_t^j)^{\gamma_t^j}}{\gamma_t^j}$) with $\gamma_2^P \geq \gamma_2^K$.
- All combinations of negative exponential utility functions.
- HARA utility functions ($u_t^j(c_t^j) = \frac{\gamma_t^j}{1-\gamma_t^j}\left(\frac{c_t^j}{\gamma_t^j} - \eta_t^j\right)^{1-\gamma_t^j}$) with $\gamma_2^K \geq \gamma_2^P$.
- All combinations of utility functions for which $u_2^K(\cdot)$ has constant absolute risk aversion.

[17]For completeness note government expenditures remain the same under each policy. Under the second policy, the government finances its expenditures by issuing a one period bond, at rate r, to some external agent.

[18]The parent's endowment must exceed the kid's endowment by an amount sufficient to ensure an operative transfer motive.

[19]For completeness I repeated the calculations for the manipulative specifications with the tax imposed on the kid in the second period. As predicted, consumption amounts and utilities were identical to those given in Table reftable:example. The only difference is an increase of $\tau(1+r) = 2.857$ in the transfer amount.

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Appendices

A Proof of Theorem 1

This appendix provides a proof that, given assumption A.1, a unique simultaneous choice equilibrium exists.

This proof relies on the contraction mapping theorem (see Stokey, Lucas, and Prescott (1989), p. 50-52). The key is to demonstrate the best response functions comprise a contraction mapping.

First recall $\mathcal{S}^j$ =[0,w^j] where w^j is individual j's ($j = P, K$) initial endowment. Let $\mathcal{S} \equiv \mathcal{S}^P \times \mathcal{S}^K$ and note $\mathcal{S}$ is a closed and bounded subset of $\mathcal{R}^2$. Thus $\mathcal{S}$ is compact and convex. For a metric I use the standard Euclidean norm,

$$\rho(x, y) = \| x - y \| = \Big(\sum_{j=1}^{2}(x_j - y_j)^2\Big)^{1/2} \quad \forall x, y \in \mathcal{S}.$$

To show a function $F(\cdot) : \mathcal{S} \to \mathcal{S}$ is a contraction we must show that for some $\beta \in (0, 1)$,

$$\rho(F(x), F(y)) \leq \beta\rho(x, y) \quad \forall x, y \in \mathcal{S}. \tag{A.1}$$

For any $s_1^P, s_2^P \in \mathcal{S}^P$ and any $s_1^K, s_2^K \in \mathcal{S}^K$, let $x = \begin{bmatrix} s_1^P \\ s_1^K \end{bmatrix}$ and $y = \begin{bmatrix} s_2^P \\ s_2^K \end{bmatrix}$.

Define $F(\cdot) = \begin{bmatrix} f^P(\cdot) \\ f^K(\cdot) \end{bmatrix}$. $f^P(\cdot)$ and $f^K(\cdot)$ are the best response functions for the parent and kid respectively, as defined in section II.B.

Then $F(x) = \begin{bmatrix} f^P(s_1^K) \\ f^K(s_1^P) \end{bmatrix}$ and $F(y) = \begin{bmatrix} f^P(s_2^K) \\ f^K(s_2^P) \end{bmatrix}$.

We first show

$$\rho(F(x), F(y)) < \rho(x, y). \tag{A.2}$$

We later show $\exists \beta \in (0, 1)$ satisfying (A.1).

Expanding equation (A.2) gives

$$\text{(A.3)} \qquad \left[(f^P(s_1^K) - f^P(s_2^K))^2 + (f^K(s_1^P) - f^K(s_2^P))^2\right]^{1/2} < \left[(s_1^P - s_2^P)^2 + (s_1^K - s_2^K)^2\right]^{1/2}.$$

That equation (A.3) is satisfied can be demonstrated by proving

$$\text{(A.4)} \qquad |f^K(s_1^P) - f^K(s_2^P)| \leq |s_1^P - s_2^P|$$

and

$$\text{(A.5)} \qquad |f^P(s_1^K) - f^P(s_2^K)| \leq |s_1^K - s_2^K|$$

$\forall (s_1^P, s_1^K), (s_2^P, s_2^K) \in \mathcal{S}$, with at least one equation satisfied with strict inequality.

Without loss of generality, assume $s_2^P > s_1^P$ and $s_2^K > s_1^K$. Then define $s_2^P = s_1^P + \delta^P$ and $s_2^K = s_1^K + \delta^K$. We proceed with proofs of equations (A.4) and (A.5) separately.

1. Given the above definition, equation (A.4) can be rewritten as

$$\text{(A.6)} \qquad |f^K(s_1^P) - f^K(s_1^P + \delta^P)| \leq \delta^P.$$

The following arguments evaluate how the kid's optimal savings choice changes when the parent's savings increases by δ^P. Therefore, s^K is a variable in the following equations.

Rewrite equation (8) as follows:

$$\text{(A.7)} \qquad T(s_1^P, s^K) = \underset{T}{\text{argmax}} \left[u_2^P(s_1^P(1+r) - T) + \rho u_2^K(s^K(1+r) + T)\right],$$

and

$$\text{(A.8)} \qquad T(s_1^P + \delta^P, s^K) = \underset{T}{\text{argmax}} \left[u_2^P((s_1^P + \delta^P)(1+r) - T) + \rho u_2^K(s^K(1+r) + T)\right].$$

Let $\Delta T = T(s_1^P + \delta^P, s^K) - T(s_1^P, s^K)$.

In equation (A.8) the parent's second period wealth is $\delta^P(1+r)$ units larger than in equation (A.7). The parent consumes some of this additional wealth and passes some of it on to the kid. Because of the strict concavity of u_2^P and u_2^K, we know he neither consumes all of it himself nor passes all of it on to his kid. Thus $0 < \Delta T < \delta^P(1+r)$.

Then rewrite equation (12):

$$f^K(s_1^P) = \underset{s^K}{\text{argmax}} \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(s_1^P, s^K))\Big]. \tag{A.9}$$

Similarly,

$$\begin{aligned} f^K(s_1^P + \delta^P) &= \underset{s^K}{\text{argmax}} \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(s_1^P + \delta^P, s^K))\Big] \\ &= \underset{s^K}{\text{argmax}} \Big[u_1^K(w^K - s^K) + \beta u_2^K(s^K(1+r) + T(s_1^P, s^K) + \Delta T)\Big]. \qquad \text{(A.10)} \end{aligned}$$

Define $\Delta s^K = f^K(s_1^P) - f^K(s_1^P + \delta^P)$ (i.e., the left-hand side of equation (A.4)). (Technically $\Delta s^K = \int_{s_1^P}^{s_1^P+\delta^P} \frac{\partial s^K}{\partial s^P} ds^P$, which exists for all utility functions satisfying assumption A.1.)

Compare the solutions of equations (A.9) and (A.10). In the latter the kid receives an additional transfer amount ΔT. He consumes some of this in the second period but, because of the strict concavity of u_1^K and u_2^K, also consumes more in the first period. Consuming more in the first period means the return from savings in the second period ($s^K(1+r)$) decreases. The decrease in the amount returned from savings is necessarily less than the additional transfer amount. Therefore,

$$\begin{aligned} &\Delta s^K(1+r) < \Delta T < \delta^P(1+r) \qquad \text{(A.11)} \\ &\implies \Delta s^K < \delta^P. \end{aligned}$$

Thus equation (A.4) holds with strict inequality.

2. We now return to equation (A.5), which can be rewritten as

$$|f^P(s_1^K) - f^P(s_1^K + \delta^K)| \leq \delta^K. \tag{A.12}$$

This section evaluates how the parent's optimal savings choice changes when the kid's savings increases by δ^K. Therefore, s^P is a variable in the following equations.

Again rewrite equation (8):

$$T(s^P, s_1^K) = \underset{T}{\operatorname{argmax}} \Big[u_2^P(s^P(1+r) - T) + \rho u_2^K(s_1^K(1+r) + T) \Big], \tag{A.13}$$

and

$$T(s^P, s_1^K + \delta^K) = \underset{T}{\operatorname{argmax}} \Big[u_2^P(s^P(1+r) - T) + \rho u_2^K((s_1^K + \delta^K)(1+r) + T) \Big]. \tag{A.14}$$

Define $\Delta T = T(s^P, s_1^K) - T(s^P, s_1^K + \delta^K)$.

In equation (A.14) the kid's second period wealth is greater by $\delta^K(1+r)$. Thus the parent reduces his transfer amount from the amount chosen in equation (A.13). Because of the strict concavity of u_2^P and u_2^K, the parent decreases his transfer amount by less than the kid's wealth increase. Thus we obtain

$$0 < \Delta T < \delta^K(1+r). \tag{A.15}$$

Next rewrite equation (11):

$$\begin{aligned} f^P(s_1^K) = \underset{s^P}{\operatorname{argmax}} \Big[& u_1^P(w^P - s^P) + \beta u_2^P(s^P(1+r) - T(s^P, s_1^K)) + \\ & \rho[u_1^K(w^K - s_1^K) + \beta u_2^K(s_1^K(1+r) + T(s^P, s_1^K))] \Big]. \end{aligned} \tag{A.16}$$

Similarly,

$$\begin{aligned} f^P(s_1^K + \delta^K) &= \underset{s^P}{\operatorname{argmax}} \Big[u_1^P(w^P - s^P) + \beta u_2^P(s^P(1+r) - T(s^P, s_1^K + \delta^K)) \\ &\quad + \rho[u_1^K(w^K - s_1^K - \delta^K) + \beta u_2^K((s_1^K + \delta^K)(1+r) + T(s^P, s_1^K + \delta^K))] \Big] \\ &= \underset{s^P}{\operatorname{argmax}} \Big[u_1^P(w^P - s^P) + \beta u_2^P(s^P(1+r) - T(s^P, s_1^K) + \Delta T) + \\ &\quad \rho[u_1^K(w^K - s_1^K - \delta^K) + \beta u_2^K((s_1^K + \delta^K)(1+r) + T(s^P, s_1^K) - \Delta T)] \Big]. \end{aligned} \tag{A.17}$$

Define $\Delta s^P = f^P(s_1^K) - f^P(s_1^K + \delta^K)$ (i.e., the left-hand side of equation (A.5)). (Technically $\Delta s^P = \int_{s_1^K}^{s_1^K + \delta^K} \frac{\partial s^P}{\partial s^K} ds^K$, which exists for all utility functions satisfying the conditions of section II.A.)

We need consider only two of the three differences between equations (A.16) and (A.17). These two are the increase (of ΔT) in the parent's second period wealth and the increase (of

$\delta^K(1+r) - \Delta T$) in the kid's second period wealth. (The third difference is the decrease (of δ^K) in the kid's first period consumption. Since the parent's transfer only directly affects the kid's second period consumption, the kid's first period consumption amount is not relevant to the parent's savings choice when taking the kid's savings amount as given.)

The *aggregate* increase in second period wealth is $\delta^K(1+r)$. The parent, via his transfer and savings decisions determines how this additional wealth will be distributed between himself and the kid. Since all utility functions are strictly concave, he distributes this wealth amongst his own first and second period consumption and the kid's second period consumption. Greater first period consumption implies less first period savings. We quantify the savings decrease by looking at the effect a savings decrease has on second period wealth. In the second period, the decreased return caused by a savings decrease ($\Delta s^P(1+r)$) must be less than the aggregate increase in second period wealth ($\delta^K(1+r)$).

Therefore,

$$\Delta s^P(1+r) < \delta^K(1+r) \qquad \text{(A.18)}$$

$$\Longrightarrow \Delta s^P < \delta^K.$$

Thus equation (A.5) holds with strict inequality.

Given equations (A.4) and (A.5) are satisfied with strict inequality then equation (A.2) is satisfied.

We now turn our attention to showing $\exists \beta \in (0,1)$ satisfying equation (A.1).

Rewriting equation (A.2) gives

$$\frac{\rho(F(x), F(y))}{\rho(x,y)} < 1. \qquad \text{(A.19)}$$

Since F and ρ are continuous, the left-hand side of equation (A.19) defines a continuous function from $\mathcal{S} \times \mathcal{S}$ to $[0,1]$.

Define β as follows:

$$\beta = \sup_{x,y \in \mathcal{S}} \frac{\rho(F(x), F(y))}{\rho(x,y)}. \qquad \text{(A.20)}$$

Note that $0 \leq \beta \leq 1$.

It is known a continuous function on a compact set achieves its supremum. That is, $\exists(\bar{x}, \bar{y}) \in \mathcal{S} \times \mathcal{S}$ such that

$$\beta = \frac{\rho(F(\bar{x}), F(\bar{y}))}{\rho(\bar{x}, \bar{y})}. \tag{A.21}$$

By way of contradiction suppose $\beta = 1$. Then $\rho(F(\bar{x}), F(\bar{y})) = \rho(\bar{x}, \bar{y})$ which contradicts equation (A.2).

Therefore it must be that $\beta < 1$ and that β satisfies equation (A.1).

Q.E.D.

B Proof of Theorem 2

This appendix provides a proof that, given assumption A.1, Ricardian equivalence holds in a static model with simultaneous first period consumption and savings choices.

Let (τ^P, τ^K) be lump-sum taxes imposed on the parent in the first period and on the kid in the second period respectively. Under the initial policy $(\tau^P, \tau^K) = (\tau, 0)$. Under the second policy $(\tau^P, \tau^K) = (0, \tau(1+r))$. To simplify notation let $u(c_t^j) = u_t^j(c_t^j)$ for $j = P, K;\ t = 1, 2$.

Start with the parent's problem. He chooses c_1^P, c_2^P, s^P and T to solve

$$\max u(c_1^P) + \beta u(c_2^P) + \rho(u(c_1^K) + \beta u(c_2^K)) \tag{B.1}$$

subject to

$$c_1^P + s^P \leq w^P - \tau^P$$

$$c_2^P + T \leq s^P(1+r)$$

$$c_2^K \leq s^K(1+r) + T - \tau^K$$

$$c_1^P, c_2^P, s^P, T \geq 0.$$

The resulting first order conditions (FOC) are

$$u'(c_1^P) - \lambda_1 \leq 0$$

$$\beta u'(c_2^P) - \lambda_2 \leq 0$$

$$-\lambda_1 + \lambda_2(1+r) \leq 0$$

$$\rho\beta u'(c_2^K) - \lambda_2 \leq 0.$$

The fact that $\lim_{c\to 0} u'(c) = \infty$ assures the first two FOC are satisfied with equality. This fact also assures a positive amount of savings since the parent has no other resources available in the second period. Thus the third FOC is satisfied with equality. Since our interest is only in cases with positive intergenerational transfers, we assume the final FOC is also satisfied with equality.

Combining the FOC gives

$$\beta(1+r)u'(c_2^P) = u'(c_1^P) \tag{B.2}$$

and

$$\rho u'(c_2^K) = u'(c_2^P). \tag{B.3}$$

Use equation (B.3) to define c_2^P in terms of c_2^K:

$$c_2^P \equiv D_2^P(c_2^K;\rho). \tag{B.4}$$

Then equation (B.2) defines c_1^P in terms of c_2^K:

$$c_1^P \equiv D_1^P(c_2^K;\rho,\beta,r). \tag{B.5}$$

Combining these with the parent's budget constraints above produces the following result.

$$w^P - \frac{T}{1+r} = D_1^P(c_2^K;\rho,\beta,r) + \frac{D_2^P(c_2^K;\rho)}{1+r} + \tau^P \tag{B.6}$$

Now consider the kid's problem. He chooses c_1^K, c_2^K and s^K to solve

$$\max u(c_1^K) + \beta u(c_2^K) \tag{B.7}$$

subject to

$$c_1^K + s^K \leq w^K$$

$$c_2^K \leq s^K(1+r) + T - \tau^K$$

$$c_1^K, c_2^K, s^K \geq 0.$$

The first order conditions for this problem are

$$u'(c_1^K) - \lambda_1 \leq 0$$

$$\beta u'(c_2^K) - \lambda_2 \leq 0$$

$$-\lambda_1 + \lambda_2(1+r+\frac{\partial T}{\partial s^K}) \leq 0. \tag{B.8}$$

Again $\lim_{c\to 0} u'(c) = \infty$ assures the first two FOC are satisfied with equality. However, since it may be optimal for the kid to choose $s^K = 0$ equation (B.8) may not hold with equality. In what follows I consider the two possible cases of strict equality and strict inequality. I show Ricardian equivalence holds in both cases.

<u>Case I:</u> Equation (B.8) satisfied with equality.

Combining the FOC gives

(B.9) $$u'(c_1^K) = \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}).$$

We separately show (in section B.1) that

(B.10) $$\frac{\partial T}{\partial s^K} = \frac{-\rho u''(c_2^K)(1+r)}{u''(c_2^P) + \rho u''(c_2^K)}.$$

Combining equation (B.4) with equation (B.10) allows us to use equation (B.9) to define c_1^K as a function of c_2^K.

(B.11) $$c_1^K \equiv D_1^K(c_2^K; \rho, \beta, r)$$

Inserting equation (B.11) into the kid's first period budget constraint gives

(B.12) $$w^K - s^K = D_1^K(c_2^K; \rho, \beta, r).$$

Combining equations (B.12) and (B.6) gives

(B.13) $$w^P - \frac{T}{1+r} + w^K - s^K = D_1^P(c_2^K; \rho, \beta, r) + \frac{D_2^P(c_2^K; \rho)}{1+r} + D_1^K(c_2^K; \rho, \beta, r) + \tau^P.$$

Rewrite the left-hand side using the kid's second period budget constraint. Then rearranging gives

(B.14) $$w^P + w^K = D_1^P(c_2^K; \rho, \beta, r) + D_1^K(c_2^K; \rho, \beta, r) + \frac{D_2^P(c_2^K; \rho) + c_2^K}{1+r} + \tau^P + \frac{\tau^K}{1+r}.$$

Use equation (B.14) to define c_2^K in terms of $(w^P + w^K - \tau^P - \frac{\tau^K}{1+r})$ and the parameters (ρ, β, r):

(B.15) $$c_2^K \equiv D_2^K((w^P + w^K - \tau^P - \frac{\tau^K}{1+r}); \rho, \beta, r).$$

Note c_2^K depends only on the sum of the initial endowments and taxes and not on their specific distribution. In addition, the value of $(-\tau^P - \frac{\tau^K}{1+r})$ is identical under both policies. Thus changing the distribution of taxes does not affect c_2^K. By extension, since the other consumption amounts are all functions of c_2^K, they also are unchanged by a change in the distribution of taxes.

Case II: Equation (B.8) not satisfied with equality.

First note this implies $s^K = 0$. Then the kid's first period budget constraint gives

(B.16)
$$c_1^K = w^K.$$

The parent's problem is unchanged from that discussed earlier, again producing equation (B.6). Combining equation (B.6) with the kid's second period budget constraint gives

(B.17)
$$w^P = D_1^P(c_2^K;\rho,\beta,r) + \frac{D_2^P(c_2^K;\rho) + c_2^K}{1+r} + \tau^P + \frac{\tau^K}{1+r}.$$

Use equation (B.17) to define c_2^K in terms of $(w^P - \tau^P - \frac{\tau^K}{1+r})$ and the parameters (ρ,β,r):

(B.18)
$$c_2^K \equiv D_2^K((w^P - \tau^P - \frac{\tau^K}{1+r});\rho,\beta,r).$$

As in Case I c_2^K depends on the sum of the taxes and not on their specific distribution. Since $(-\tau^P - \frac{\tau^K}{1+r})$ is identical under both policies, changing the distribution of taxes does not affect c_2^K. By extension c_1^P and c_2^P are also unaffected.

To show c_1^K is unaffected by a change in the timing of taxes consider equation (B.8), rewritten here after substituting in the other first order conditions.

(B.19)
$$-u'(c_1^K) + \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}) < 0$$

The second term is unchanged by the change in tax policy. Since the kid cannot borrow, c_1^K cannot increase, but could decrease. By way of contradiction suppose c_1^K decreases. The first term in equation (B.19) becomes more negative, and the left-hand side remains strictly negative. By the Kuhn-Tucker conditions we still get $s^K = 0$ and therefore $c_1^K = w^K$. Thus it must be c_1^K is unaffected by the change in the distribution of taxes.

Q.E.D.

B.1 Derivation of Equation (B.10)

First recall the following equation, developed in section II.A. This equation provides the parent's optimal second period transfer choice given the savings decisions of the parent and kid.

$$T(s^P, s^K) \equiv \underset{T}{\operatorname{argmax}} \left[u_2^P(s^P(1+r) - T) + \rho u_2^K(s^K(1+r) + T - \tau^K)\right] \tag{B.20}$$

such that $T \geq 0$.

Differentiate the expression within brackets on the right-hand side of equation (B.20) with respect to T to get

$$-u'(s^P(1+r) - T) + \rho u'(s^K(1+r) + T - \tau^K) = 0. \tag{B.21}$$

Differentiating this with respect to s^K gives

$$u''(s^P(1+r) - T)\frac{\partial T}{\partial s^K} + \rho u''(s^K(1+r) + T - \tau^K)(1 + r + \frac{\partial T}{\partial s^K}) = 0. \tag{B.22}$$

Solve for $\frac{\partial T}{\partial s^K}$:

$$\frac{\partial T}{\partial s^K} = \frac{-\rho u''(c_2^K)(1+r)}{u''(c_2^P) + \rho u''(c_2^K)}.$$

which is the expression of equation (B.10).

C Proof of Theorem 3

This section presents a proof that, given assumption A.1, Ricardian equivalence holds in the static model when the parent chooses first in the first period.

Let (τ^P, τ^K) be lump sum taxes imposed on the parent in the first period and on the kid in the second period respectively. Under the initial policy $(\tau^P, \tau^K) = (\tau, 0)$. Under the second policy $(\tau^P, \tau^K) = (0, \tau(1+r))$. To simplify notation let $u(c_t^j) = u_t^j(c_t^j)$ for $j = P, K;\ t = 1, 2$.

The kid's problem is identical to that presented in Appendix B for the simultaneous choice specification. He chooses consumption and savings taking the parent's savings amount as given. There were two cases for the kid's FOC discussed in appendix B. Both are considered here as well.

Case I: $-u'(c_1^K) + \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}) = 0.$

From appendix B, the kid's FOC again reduce to

(C.1) $$u'(c_1^K) = \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}),$$

where $\partial T / \partial s^K$ is unchanged from before.

Again define c_1^K as a function of c_2^K using equation (C.1):

(C.2) $$c_1^K \equiv D_1^K(c_2^K; \rho, \beta, r).$$

Inserting this expression into the kid's first period budget constraint gives

(C.3) $$w^K - s^K = D_1^K(c_2^K; \rho, \beta, r).$$

Now consider the parent's problem. He chooses c_1^P, c_2^P, s^P and T to solve

(C.4) $$\max u(c_1^P) + \beta u(c_2^P) + \rho(u(c_1^K) + \beta u(c_2^K))$$

subject to

$$c_1^P + s^P \leq w^P - \tau^P$$

$$c_2^P + T \leq s^P(1+r)$$

$$c_1^K + s^K \leq w^K$$

$$c_2^K \leq s^K(1+r) + T - \tau^K$$

$$c_1^P, c_2^P, s^P, T \geq 0.$$

The resulting first order conditions are

$$u'(c_1^P) - \lambda_1 \leq 0$$

$$\beta u'(c_2^P) - \lambda_2 \leq 0$$

$$-\lambda_1 + \lambda_2(1+r) - \rho u'(c_1^K)\frac{\partial s^K}{\partial s^P} + \rho\beta u'(c_2^K)(1+r)\frac{\partial s^K}{\partial s^P} \leq 0$$

$$\rho\beta u'(c_2^K) - \lambda_2 \leq 0.$$

The fact that $\lim_{c \to 0} u'(c) = \infty$ assures the first two FOC are satisfied with equality. This fact also assures a positive savings amount since the parent has no other resources available in the second period. Thus the third FOC is satisfied with equality. Since our interest is only in cases with positive intergenerational transfers, we assume the final FOC is satisfied with equality.

Combining the FOC gives

$$-u'(c_1^P) + \beta(1+r)u'(c_2^P) + \rho\frac{\partial s^K}{\partial s^P}\left[-u'(c_1^K) + \beta(1+r)u'(c_2^K)\right] = 0 \tag{C.5}$$

and

$$\rho u'(c_2^K) = u'(c_2^P). \tag{C.6}$$

Equation (C.6) defines c_2^P in terms of c_2^K:

$$c_2^P \equiv D_2^P(c_2^K; \rho). \tag{C.7}$$

Next we need to define c_1^P solely as a function of c_2^K. To accomplish this we first need to find $\frac{\partial s^K}{\partial s^P}$. For this we note equation (C.1) is the kid's first order condition for savings (s^K).

Differentiating it with respect to s^P and rearranging gives

$$\text{(C.8)} \qquad \frac{\partial s^K}{\partial s^P} = \frac{-A\beta(1+r)^2}{A\beta(1+r)^2 + u''(c_1^K)(u''(c_2^P) + \rho u''(c_2^K))^3}$$

where

$$A = \Big[\rho u'(c_2^K)\big(\rho u'''(c_2^P)[u''(c_2^K)]^2 - u'''(c_2^K)[u''(c_2^P)]^2\big) + u''(c_2^K)[u''(c_2^P)]^2(u''(c_2^P) + \rho u''(c_2^K))\Big]$$

(See section C.1 for details of this derivation.)

Combining equation (C.8) with equations (C.5) and (C.7), we can define c_1^P in terms of c_2^K:

$$\text{(C.9)} \qquad c_1^P \equiv D_1^P(c_2^K; \rho, \beta, r).$$

Combining this and equation (C.7) with the parent's budget constraints produces the following result:

$$\text{(C.10)} \qquad w^P - \frac{T}{1+r} = D_1^P(c_2^K; \rho, \beta, r) + \frac{D_2^P(c_2^K; \rho)}{1+r} + \tau^P.$$

Combining this with equation (C.3) gives

$$\text{(C.11)} \qquad w^P - \frac{T}{1+r} + w^K - s^K = D_1^P(c_2^K; \rho, \beta, r) + \frac{D_2^P(c_2^K; \rho)}{1+r} + D_1^K(c_2^K; \rho, \beta, r) + \tau^P.$$

Rewriting the left-hand side using the kid's second period budget constraint, and rearranging, gives

$$\text{(C.12)} \qquad w^P + w^K = D_1^P(c_2^K; \rho, \beta, r) + D_1^K(c_2^K; \rho, \beta, r) + \frac{D_2^P(c_2^K; \rho) + c_2^K}{1+r} + \tau^P + \frac{\tau^K}{1+r}.$$

Use equation (C.12) to define c_2^K in terms of $(w^P + w^K - \tau^P - \frac{\tau^K}{1+r})$ and the parameters (ρ, β, r).

$$\text{(C.13)} \qquad c_2^K \equiv D_2^K((w^P + w^K - \tau^P - \frac{\tau^K}{1+r}); \rho, \beta, r).$$

Note c_2^K depends only on the sum of the initial endowments and taxes and not on their specific distribution. In addition, the value of $(-\tau^P - \frac{\tau^K}{1+r})$ is identical under both policies. Thus changing the distribution of taxes does not affect c_2^K. By extension, since the other consumption amounts are all functions of c_2^K, they also are unchanged by a change in the distribution of taxes.

Case II: $-u'(c_1^K) + \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}) < 0$.

First note this implies $s^K = 0$. Then the kid's first period budget constraint gives

$$c_1^K = w^K. \tag{C.14}$$

The parent's problem is still the same as that described for Case I.

Proposition: When $-u'(c_1^K) + \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}) < 0$ then $\frac{\partial s^K}{\partial s^P} = 0$.

Proof: First note $\frac{\partial T}{\partial s^P} > 0$, as shown in section C.2. Thus T increases when s^P increases. As argued in appendix A the kid decreases his savings amount when T increases. Thus the kid desires to decrease his savings amount when s^P increases. However, since the kid's savings amount is already zero, he cannot save less. Thus his savings do not change and $\frac{\partial s^K}{\partial s^P} = 0$. ■

Given $\frac{\partial s^K}{\partial s^P} = 0$ the parent's first order conditions reduce to

$$\rho u'(c_2^K) - u'(c_2^P) \leq 0 \tag{C.15}$$

and

$$-u'(c_1^P) + \beta(1 + r)u'(c_2^P) \leq 0. \tag{C.16}$$

The fact that $\lim_{c\to 0} u'(c) = \infty$ assures equation (C.16) is satisfied with equality. Since we are interested only in cases with positive intergenerational transfers, we assume equation (C.15) is satisfied with equality.

Equation (C.15) defines c_2^P in terms of c_2^K:

$$c_2^P \equiv D_2^P(c_2^K; \rho). \tag{C.17}$$

Then equation (C.16) defines c_1^P in terms of c_2^K:

$$c_1^P \equiv D_1^P(c_2^K; \rho, \beta, r). \tag{C.18}$$

Combining these with the parent's budget constraints produces

$$w^P - \frac{T}{1+r} = D_1^P(c_2^K;\rho,\beta,r) + \frac{D_2^P(c_2^K;\rho)}{1+r} + \tau^P. \tag{C.19}$$

Combine this with the kid's second period budget constraint to get

$$w^P = D_1^P(c_2^K;\rho,\beta,r) + \frac{D_2^P(c_2^K;\rho) + c_2^K}{1+r} + \tau^P + \frac{\tau^K}{1+r}. \tag{C.20}$$

Use equation (C.20) to define c_2^K in terms of $(w^P - \tau^P - \frac{\tau^K}{1+r})$ and the parameters (ρ,β,r):

$$c_2^K \equiv D_2^K((w^P - \tau^P - \frac{\tau^K}{1+r});\rho,\beta,r). \tag{C.21}$$

As in Case I, c_2^K depends on the sum of the taxes and not on their specific distribution. Since $(-\tau^P - \frac{\tau^K}{1+r})$ is identical under both policies, changing the distribution of taxes does not affect c_2^K. By extension c_1^P and c_2^P are also unaffected.

To show c_1^K is unaffected by a change in the timing of taxes consider equation (B.8), rewritten here after substituting in the other first order conditions.

$$-u'(c_1^K) + \beta u'(c_2^K)(1 + r + \frac{\partial T}{\partial s^K}) < 0 \tag{C.22}$$

The second term is unchanged by the change in tax policy. Since the kid cannot borrow, c_1^K cannot increase, but could decrease. By way of contradiction suppose c_1^K decreases. The first term in equation (C.22) becomes more negative, and the left-hand side remains strictly negative. By the Kuhn-Tucker conditions we still get $s^K = 0$ and therefore $c_1^K = w^K$. Thus it must be c_1^K is unaffected by the change in the distribution of taxes.

Q.E.D.

C.1 Derivation of Equation (C.8)

Equation (C.1) gives the kid's first order condition for savings. Differentiate it with respect to s^P to get:

$$u''(c_1^K)\Big(-\frac{\partial s^K}{\partial s^P}\Big) = \beta u''(c_2^K)\Big(\frac{\partial T}{\partial s^P}\Big)^2\Big(1+\frac{\partial s^K}{\partial s^P}\Big) + \left[\frac{\beta u'(c_2^K)(1+r)(1+\frac{\partial s^K}{\partial s^P})}{\Big(u''(c_2^P)+\rho u''(c_2^K)\Big)^2}\right]\times$$

$$\left[\Big(u''(c_2^P)+\rho u''(c_2^K)\Big)u'''(c_2^P)\Big(-\frac{\partial T}{\partial s^K}\Big) - u''(c_2^P)\Big(u'''(c_2^P)\Big(-\frac{\partial T}{\partial s^K}\Big)+\rho u'''(c_2^K)\frac{\partial T}{\partial s^P}\Big)\right]. \tag{C.23}$$

Rearranging to solve for $\partial s^K/\partial s^P$ gives

$$\frac{\partial s^K}{\partial s^P} = \frac{-A\beta(1+r)^2}{A\beta(1+r)^2 + u''(c_1^K)(u''(c_2^P)+\rho u''(c_2^K))^3} \tag{C.24}$$

where

$$A = \Big[\rho u'(c_2^K)\big(\rho u'''(c_2^P)[u''(c_2^K)]^2 - u'''(c_2^K)[u''(c_2^P)]^2\big) + u''(c_2^K)[u''(c_2^P)]^2(u''(c_2^P)+\rho u''(c_2^K))\Big]$$

which is the expression of equation (C.8). It is easily shown that $\partial s^K/\partial s^P$ exists for all utility functions satisfying assumption A.1.

C.2 Derivation of $\frac{\partial T}{\partial s^P}$

First recall the following equation, developed in section II.A. This equation provides the parent's optimal second period transfer choice as a function of the savings decisions of the parent and kid.

$$T(s^P, s^K) \equiv \underset{T}{\text{argmax}} \Big[u_2^P(s^P(1+r)-T) + \rho u_2^K(s^K(1+r)+T-\tau^K)\Big] \tag{C.25}$$

$$\text{such that } T \geq 0.$$

Differentiate the expression within brackets on the right-hand side of equation (C.25) with respect to T to get

$$-u'(s^P(1+r)-T) + \rho u'(s^K(1+r)+T-\tau^K) = 0. \tag{C.26}$$

Differentiate equation (C.26) with respect to s^P (holding s^K constant) to get

$$-u''(c_2^P)\left[1 + r - \frac{\partial T}{\partial s^P}\right] + \rho u''(c_2^K)\frac{\partial T}{\partial s^P} = 0. \tag{C.27}$$

Solve for $\frac{\partial T}{\partial s^P}$:

$$\frac{\partial T}{\partial s^P} = \frac{u''(c_2^P)(1+r)}{u''(c_2^P) + \rho u''(c_2^K)} \tag{C.28}$$

By assumption $u(\cdot)$ is strictly concave so $u''(\cdot) < 0$.

Thus $\frac{\partial T}{\partial s^P} > 0$.

D Proof of Theorem 4

This section presents a proof that, given assumption A.1, Ricardian equivalence holds in the static model when the kid chooses first in the first period.

Let (τ^P, τ^K) be lump sum taxes imposed on the parent in the first period and on the kid in the second period respectively. Under the initial policy $(\tau^P, \tau^K) = (\tau, 0)$. Under the second policy $(\tau^P, \tau^K) = (0, \tau(1+r))$. To simplify notation let $u(c_t^j) = u_t^j(c_t^j)$ for $j = P, K;\ t = 1, 2$.

Start with the parent's problem, which is identical to that presented in Appendix B for the simultaneous choice specification. He chooses consumption and savings taking the kid's savings choice as given. Thus his first order conditions again reduce to

$$\beta(1+r)u'(c_2^P) = u'(c_1^P) \tag{D.1}$$

and

$$\rho u'(c_2^K) = u'(c_2^P). \tag{D.2}$$

These FOC again define c_2^P and c_1^P in terms of c_2^K (repeated here for convenience.)

$$c_2^P \equiv D_2^P(c_2^K; \rho) \tag{D.3}$$

$$c_1^P \equiv D_1^P(c_2^K; \rho, \beta, r) \tag{D.4}$$

Combining these with the parent's budget constraints produces the following result:

$$w^P - \frac{T}{1+r} = D_1^P(c_2^K; \rho, \beta, r) + \frac{D_2^P(c_2^K; \rho)}{1+r} + \tau^P. \tag{D.5}$$

Additionally, note equation (D.1) is the parent's first order condition for savings (s^P). Differentiate it with respect to s^K to find $\partial s^P / \partial s^K$.

$$u''(c_1^P)\frac{\partial s^P}{\partial s^K} + \beta u''(c_2^P)(1+r)\left[(1+r)\frac{\partial s^P}{\partial s^K} - \frac{\partial T}{\partial s^K} - \frac{\partial T}{\partial s^P}\frac{\partial s^P}{\partial s^K}\right] = 0 \tag{D.6}$$

Rearranging, and using the fact that $(1+r)+\frac{\partial T}{\partial s^K}=\frac{\partial T}{\partial s^P}$ (see section D.1) gives

$$\frac{\partial s^P}{\partial s^K}=\frac{\beta u''(c_2^P)(1+r)\frac{\partial T}{\partial s^K}}{u''(c_1^P)-\beta u''(c_2^P)(1+r)\frac{\partial T}{\partial s^K}}. \tag{D.7}$$

We also show in section D.1 that

$$\frac{\partial T}{\partial s^K}=\frac{-\rho u''(c_2^K)(1+r)}{u''(c_2^P)+\rho u''(c_2^K)}. \tag{D.8}$$

Given this, and equations (D.3) and (D.4), $\frac{\partial s^P}{\partial s^K}$ can be expressed as a function of c_2^K.

Now consider the kid's problem. He chooses c_1^K, c_2^K and s^K to solve

$$\max u(c_1^K)+\beta u(c_2^K) \tag{D.9}$$

subject to

$$c_1^K+s^K\leq w^K$$

$$c_2^K\leq s^K(1+r)+T-\tau^K$$

$$c_1^K, c_2^K, s^K\geq 0.$$

The first order conditions for this problem are

$$u'(c_1^K)-\lambda_1\leq 0$$

$$\beta u'(c_2^K)-\lambda_2\leq 0$$

$$-\lambda_1+\lambda_2\left[1+r+\frac{\partial T}{\partial s^K}+\frac{\partial T}{\partial s^P}\frac{\partial s^P}{\partial s^K}\right]\leq 0. \tag{D.10}$$

Again $\lim_{c\to 0}u'(c)=\infty$ assures the first two FOC are satisfied with equality. However, since it may be optimal for the kid to choose $s^K=0$ equation (D.10) may not hold with equality. In what follows I consider the two possible cases of strict equality and strict inequality. I show Ricardian equivalence holds in both cases.

<u>Case I:</u> Equation (D.10) satisfied with equality.

Combining the FOC, and again using the fact that $(1+r)+\frac{\partial T}{\partial s^K}=\frac{\partial T}{\partial s^P}$ gives

$$u'(c_1^K)=\beta u'(c_2^K)(1+\frac{\partial s^P}{\partial s^K})\frac{\partial T}{\partial s^P}. \tag{D.11}$$

We separately show (see section D.1) that

$$\frac{\partial T}{\partial s^P}=\frac{u''(c_2^P)(1+r)}{u''(c_2^P)+\rho u''(c_2^K)}. \tag{D.12}$$

Given equation (D.3), use equation (D.12) to define

$$\frac{\partial T}{\partial s^P}\equiv D^T(c_2^K;\rho,r) \tag{D.13}$$

Combining this with equations (D.3), (D.4) and (D.7) allows use of equation (D.11) to define c_1^K as a function of c_2^K.

$$c_1^K\equiv D_1^K(c_2^K;\rho,\beta,r) \tag{D.14}$$

Inserting this expression into the kid's first period budget constraint gives

$$w^K-s^K=D_1^K(c_2^K;\rho,\beta,r). \tag{D.15}$$

Combining equation (D.15) with equation (D.5) gives

$$w^P-\frac{T}{1+r}+w^K-s^K=D_1^P(c_2^K;\rho,\beta,r)+\frac{D_2^P(c_2^K;\rho)}{1+r}+D_1^K(c_2^K;\rho,\beta,r)+\tau^P. \tag{D.16}$$

Rewriting the left-hand side using the kid's second period budget constraint, and rearranging, gives

$$w^P+w^K=D_1^P(c_2^K;\rho,\beta,r)+D_1^K(c_2^K;\rho,\beta,r)+\frac{D_2^P(c_2^K;\rho)+c_2^K}{1+r}+\tau^P+\frac{\tau^K}{1+r}. \tag{D.17}$$

Use equation (D.17) to define c_2^K in terms of $(w^P+w^K-\tau^P-\frac{\tau^K}{1+r})$ and the parameters (ρ,β,r).

$$c_2^K\equiv D_2^K((w^P+w^K-\tau^P-\frac{\tau^K}{1+r});\rho,\beta,r) \tag{D.18}$$

Note c_2^K depends only on the sum of the initial endowments and taxes and not on their specific distribution. In addition, the value of $(-\tau^P - \frac{\tau^K}{1+r})$ is identical under both policies. Thus changing the distribution of taxes does not affect c_2^K. By extension, since the other consumption amounts are all functions of c_2^K, they also are unchanged by a change in the distribution of taxes.

<u>Case II:</u> Equation (D.10) not satisfied with equality.

First note this implies $s^K = 0$. Then the kid's first period budget constraint gives

(D.19) $$c_1^K = w^K.$$

The parent's problem is unchanged from that studied earlier, again producing equation (D.5). Combining equation (D.5) with the kid's second period budget constraint gives

(D.20) $$w^P = D_1^P(c_2^K;\rho,\beta,r) + \frac{D_2^P(c_2^K;\rho) + c_2^K}{1+r} + \tau^P + \frac{\tau^K}{1+r}.$$

Use equation (D.20) to define c_2^K in terms of $(w^P - \tau^P - \frac{\tau^K}{1+r})$ and the parameters (ρ,β,r):

(D.21) $$c_2^K \equiv D_2^K((w^P - \tau^P - \frac{\tau^K}{1+r});\rho,\beta,r).$$

As in Case I, c_2^K depends on the sum of the taxes and not on their specific distribution. Since $(-\tau^P - \frac{\tau^K}{1+r})$ is identical under both policies, changing the distribution of taxes does not affect c_2^K. By extension c_1^P and c_2^P are also unaffected.

To show c_1^K is unaffected by a change in the timing of taxes consider equation (D.10), rewritten here after substituting in the other first order conditions.

(D.22) $$-u'(c_1^K) + \beta u'(c_2^K)\left[1 + r + \frac{\partial T}{\partial s^K} + \frac{\partial T}{\partial s^P}\frac{\partial s^P}{\partial s^K}\right] < 0$$

Each component of the second term has been reduced to a function of c_2^K and thus is unchanged by the change in tax policy. Since the kid cannot borrow, c_1^K cannot increase, but could decrease.

By way of contradiction suppose c_1^K decreases. The first term in equation (D.22) becomes more negative, and the left-hand side remains strictly negative. By the Kuhn-Tucker conditions we still get $s^K = 0$ and therefore $c_1^K = w^K$. Thus it must be c_1^K is unaffected by the change in the distribution of taxes.

Q.E.D.

D.1 Derivation of Equation (D.12)

First recall the following equation, developed in section II.A, which provides the parent's optimal second period transfer choice given the savings decisions of the parent and kid.

$$T(s^P, s^K) \equiv \underset{T}{\text{argmax}} \left[u_2^P(s^P(1+r) - T) + \rho u_2^K(s^K(1+r) + T - \tau^K)\right] \tag{D.23}$$

$$\text{such that } T \geq 0.$$

Differentiate the expression within brackets on the right-hand side of equation (D.23) with respect to T to get

$$-u'(s^P(1+r) - T) + \rho u'(s^K(1+r) + T - \tau^K) = 0. \tag{D.24}$$

Differentiate equation (D.24) with respect to s^P (holding s^K constant) to get

$$-u''(c_2^P)\left[1 + r - \frac{\partial T}{\partial s^P}\right] + \rho u''(c_2^K)\frac{\partial T}{\partial s^P} = 0. \tag{D.25}$$

Solve for $\frac{\partial T}{\partial s^P}$:

$$\frac{\partial T}{\partial s^P} = \frac{u''(c_2^P)(1+r)}{u''(c_2^P) + \rho u''(c_2^K)}$$

which is the expression of equation (D.12).

Next differentiate (D.24) with respect to s^K giving

$$-u''(c_2^P)\left[(1+r)\frac{\partial s^P}{\partial s^K} - \frac{\partial T}{\partial s^K} - \frac{\partial T}{\partial s^P}\frac{\partial s^P}{\partial s^K}\right] + \rho u''(c_2^K)\left[1 + r + \frac{\partial T}{\partial s^K} + \frac{\partial T}{\partial s^P}\frac{\partial s^P}{\partial s^K}\right] = 0. \tag{D.26}$$

Apply the envelope theorem and solve for $\frac{\partial T}{\partial s^K}$:

(D.27)
$$\frac{\partial T}{\partial s^K} = \frac{-\rho u''(c_2^K)(1+r)}{u''(c_2^P) + \rho u''(c_2^K)}.$$

Combining the above results reveals

(D.28)
$$1 + r + \frac{\partial T}{\partial s^K} = \frac{\partial T}{\partial s^P}.$$